Solutions guideline for MA Comprehensive Examination (August 2008)

These are not definitive solutions. For several questions, there are multiple acceptable answers. These should be interpreted as a guideline on how to go about solving the problems.

Short questions

Question 1:

The pure strategy Nash equilibria are (T,M) and (B,R).

Question 2:

i. Decreasing returns to scale occurs when for $a > 1 \Rightarrow f(ak, al) < af(k, l)$. Equivalently, doubling all inputs leads to a less-than-doubling of output.
ii. Increasing returns to scale occurs when for $a > 1 \Rightarrow f(ak, al) > af(k, l)$.

Question 3:

For δ sufficiently close to 1, defect is not a dominant strategy. That's because it is not a best response to the strategy: play C until the opponent plays D, at which point always play D.

Question 4:

a. Rock-paper-scissors is *not* a game of perfect information because not every decision node is a singleton. It does not have a PSNE.

b. The adapted form of chess is a finite game of perfect information. Therefore there exists a PSNE which can be evaluated by backwards induction.

Question 5:

a. Because the suppliers are rationed; they are heterogeneous in MC of labor supply and so we don't know what the producer surplus is (because we don't know which ones are employed).

b. Answers:

   i. The best case scenario is that the 500 units with the lowest marginal costs of producing labor are the 500 employed.
   ii. The worst case scenario is that the 500 units with the highest marginal costs below $7.00 are the 500 employed. [Credit should be given if one argues that the guys with low MC will buy the rights to work under the rationing].

Question 6:

a. A preferred formulation is that the Coase theorem says the following: When transaction costs are negligible, parties achieve an efficient outcome.

b. The Coase theorem does not apply to LA air pollution because the transaction costs of internalizing effects are not negligible

c. The costs of internalizing an effect are a kind of transaction costs. When those are high, the effect is not internalized, and we call the effect an externality. In the case of air pollution, the high transaction costs are the costs of affected parties organizing voluntarily to reduce air pollution, or, alternatively, the costs of mediating the grievance as a tort. Either way, the transaction costs of resolving the matter are high.

Question 7:

This industry has increasing returns to scale. Therefore only one firm can operate.

Question 8:

a. If you take an umbrella, your utility is -3 for sure. If you do not, your utility is $(0)\left(\frac{4}{5}\right) + (-10)\left(\frac{1}{5}\right) = -2$. Therefore do not carry an umbrella.

b. If you get the report and it turns out to be high pressure, then you choose not to carry the umbrella and your utility is $(0)\left(\frac{7}{8}\right) + (-10)\left(\frac{1}{8}\right) = -\frac{5}{4}$. If you get the report and it turns out to be low pressure, then you choose to carry the umbrella since if you do not, your utility is $(0)\left(\frac{1}{2}\right) + (-10)\left(\frac{1}{2}\right) = -5 < -3$. Therefore your unconditional expected utility of getting the report is $\left(\frac{4}{5}\right)\left(-\frac{5}{4}\right) + \left(\frac{1}{5}\right)(-3) = -\frac{8}{5}$. Since the utility of not getting the report is -2, you are willing to pay $\left(-\frac{8}{5}\right) - (-2) = \$0.40$ for it.

Question 9:

This is the matrix of utilities:

Activity	Tom	Joe	Abe	Pam	Sue	**Total**
Home	0	0	0	0	0	**0**
Club	14-20 = -6	14-20 = -6	14-20 = -6	26	40	**48**
Movie	60-14 = 46	20-14 = 6	20-14 = 6	10-14 = -4	0-14 = -14	**40**

a. Going to the club generates the highest surplus and is therefore the most efficient.

b. In pair-wise voting, it loses to both home and movie because all three men would vote against (the women would vote in favor).

c. If Sue was to offer $7 to each of the men, everyone would benefit from going to the club relative to staying at home. There is an infinity of acceptable transfers.

Question 10:

a. The minimum amount Sam would accept to supply the 7th hour is $4.58
b. Answer:
   i. Sam's MC of the 8th hour must be greater (or equal to) $4.75.
   ii. Sam's MC of the 6th hour must be less than (or equal to) $4.75.
c. Answer:
   i. The maximum amount she'd be willing to pay for the 7th hour is $7.23.
   ii. The social surplus of transaction the 7th unit is $2.65, which is the amount by which the marginal benefit exceeds the marginal cost.

Long questions

Question 1:

a. These two goods are perfect substitutes. The solution is $s^* = \begin{cases} 0 \; if \frac{\alpha}{\beta} < \frac{p_s}{p_c} \\ \frac{m}{p_s} \; if \; \frac{\alpha}{\beta} \geq \frac{p_s}{p_c} \end{cases}, c^* = \begin{cases} \frac{m}{p_c} \; if \frac{\alpha}{\beta} < \frac{p_s}{p_c} \\ 0 \; if \; \frac{\alpha}{\beta} \geq \frac{p_s}{p_c} \end{cases}$. Clearly neither will be inferior since you are always spending either all your money on one good or none of it. Diagrammatic arguments are acceptable.

b. A horizontal line at $y = -1$ that jumps up to $y = 0$ at the point $p_s = \frac{\alpha p_c}{\beta}$.

c. No, since by non-satiation I will always spend my entire income. That means that at least one of the goods has to be normal. Inferiority is a necessary condition for being a Giffen good.

d. According to consumer theory, normal goods must obey the law of demand and Giffen goods must be inferior. These are two testable predictions: if you find normal goods you can vary the price to confirm that they are ordinary. If you find Giffen goods, you can vary income to confirm that they are inferior.

Question 2:

a. The NE is for everyone to contribute zero . The marginal utility of contributing anything is always $\beta - 1 < 0$.

b. If $N\beta > 1$ then maximal contributions by everyone Pareto dominate the NE.

c. Each i solves: $\max_{x_i \in [0,e]} \left\{ e - x_i + \beta\left(\sum_{j=1}^{N} x_j\right) + R\left(\frac{x_i}{\sum_{j=1}^{N} x_j}\right)\right\}$. The FOC is: $\beta - 1 + R\left(\frac{\sum_{j\neq i}^{N} x_j}{\left(\sum_{j=1}^{N} x_j\right)^2}\right) = 0$. Under a symmetric NE, $x_i = x^* \; \forall \; i$ so $x^* = \frac{(N-1)R}{N^2(1-\beta)}$.

d. The FOC is unchanged. Therefore total net contributions will be $Nx^* - R = \frac{(N-1)R}{N(1-\beta)} - R = \frac{R}{N(1-\beta)}\left((N-1) - N(1-\beta)\right) = \frac{R(N\beta - 1)}{N(1-\beta)} > 0$.

e. This is a public goods problem. Contributing to charity has a positive externality and will therefore be underprovided. Lotteries have negative externalities and will therefore be overprovided. These two externalities can cancel each other out.

Question 3:

a. The marginal utility to i of increasing own effort is $e_j - 2e_i$. The first-order condition (which is clearly sufficient here) is $e_i^* = \frac{1}{2}e_j^*$. Therefore the NE is $e_1^* = e_2^* = 0$.

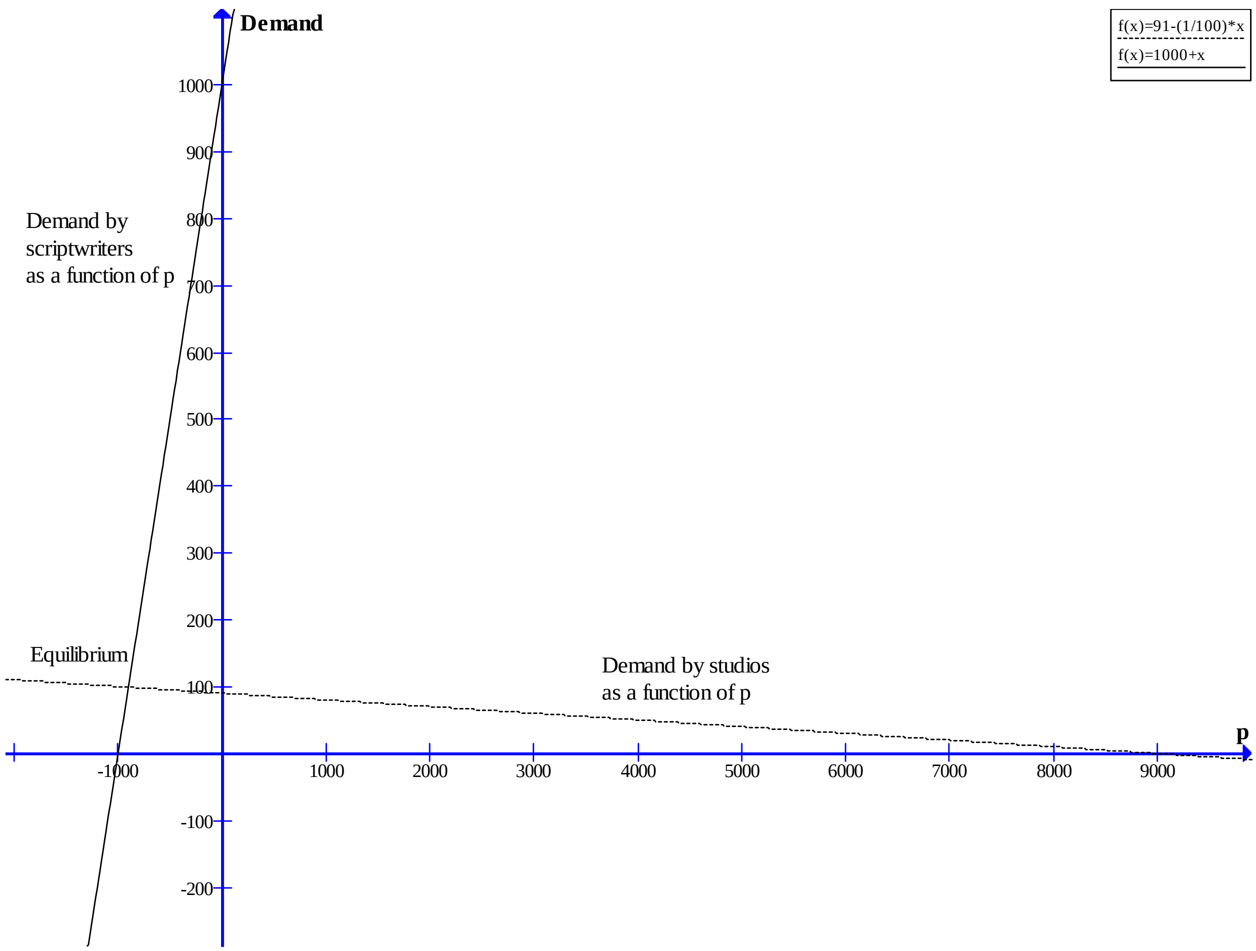
b. The strongest possible sanction is infinite zero effort in response to the first defection. The utility of infinite effort of k for each player is $\frac{k}{1-\delta}$. By the FOC from part (a), a defection by i involves playing $e_i = \frac{1}{2}k$ which yields a utility of $\frac{1}{4}\alpha k^2 + k$ for that period and zero thereafter. The smallest δ that can sustain this is obtained by solving $\frac{1}{4}\alpha k^2 + k = \frac{k}{1-\delta}$. Since $k > 0$, this can be rewritten $\alpha k + 4 = \frac{4}{1-\delta}$ and so $\delta = \frac{\alpha k}{\alpha k+4}$.

c. The utilitarian planner with equal weights solves the problem $\max_{e_1 \geq 0, e_2 \geq 0}\{\alpha e_1(e_2 - e_1) + e_2 + \alpha e_2(e_1 - e_2) + e_1\}$. Insisting on a symmetric solutions simplifies the problem to $\max_e\{2e\}$. Clearly the solution is infinite effort.

d. Lots of valid reasons. Infinite dynasties: organizations continue in perpetuity and "play" indefinitely, even if individuals within the organization do not. The classic example is a family where each generation cares about the ones after it. This model can also represent a constant probability $(1 - \delta)$ of the game ending every period. Infinite horizons can approximate long horizons under bounded rationality. Anything sensible.

Question 4:

a. Note that both prices can be negative. The equilibrium is obtained by solving $91 - \frac{1}{100}p = 1000 - q$ subject to $p = -q$. This yields $p^* = -900$, which means that the scriptwriters are paying the studios \$900 each to produce their scripts. The equilibrium trades will be 100 movies (scripts). Total welfare is the sum of the triangle with vertices where each demand meets the price axis and the equilibrium (see diagram below; note the inverted axes as compared to normal demand and supply diagrams). The base of the triangle is $(1000 + 9100) = 10100$. The height is 100 and so the total welfare is $50 \times 10100 = \$505000$.



b. As you recall from undergrad micro, it doesn't matter who bears the legislative burden of the tax when it comes to calculating the economic burden. Let's model the studios as legally paying the tax. Their demand function becomes $D = 41 - \frac{1}{100}p$ (to get this, rearrange so that p is the subject then subtract 500 from the RHS and rearrange back). This implies $p_t^* = -950$ and the equilibrium trades is 50. Studios are bearing \$4950 of the tax while scriptwriters bear \$50. This is expected since the demand of scriptwriters is very elastic compared to that of the studios. The area of the welfare triangle is now $(1000 + 4100) \times 50 \times \frac{1}{2} = \$127500$. Therefore the decrease in welfare is $\$505000 - \$127500 = \$377500$. However taxes collected are $\$5000 \times 50 = \$250000$ and so the net decrease in welfare is $\$127500$.

c. If the price is constrained to be 0, 91 studios and 1000 scriptwriters will show up to stand in line. That means that 91 movies will be made.

d. Again the price is constrained to be 0. The welfare to studios is the area under their demand from price 0 to price 9100, which is 414050. The welfare to writers is the value of the average writer who has a positive demand at price 0 times 91. The writers' values are distributed uniformly from 1000 to 0, so the average guy has a value of 500. Therefore the writer surplus is 45500, and so total surplus is 459550. Therefore the decrease in surplus compared to part (a) is \$45450.